

How Does Income Inequality Affect Market Outcomes in Vertically Differentiated Markets? *

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Abstract

The distribution of consumer incomes is a key factor in determining the structure of a vertically differentiated industry when consumer's willingness to pay depends on her income. This paper computes the Shaked and Sutton (1982) model for a lognormal distribution of consumer incomes to investigate the effect of inequality on firms' entry, product quality, and pricing decisions. The main findings are that greater inequality in consumer incomes leads to the entry of more firms and results in more intense quality competition among the entrants. More intense quality competition raises the average quality of products in the market as firms compete for the shrinking share of higher income consumers. With zero costs of quality improvements and an upper bound on the top quality or when costs of quality are fixed and rise sufficiently fast, greater heterogeneity of consumer incomes also reduces firms' incentives to differentiate their products. Competition between more similar products tends to reduce their prices. However, when income inequality is very high, the top quality producer chooses to serve only the rich segment of the market and charges a higher price. The conclusion is that income inequality has important implications for the degree of product differentiation, price level, industry concentration, and consumer welfare.

Keywords: vertical differentiation, income inequality, computational economics

JEL classification: L13, L11, C61

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1 Introduction

In this paper I study decisions of firms operating in a vertically differentiated market. The products offered in such a market differ in quality. The consumers are perfectly informed of the products' characteristics and have the same ranking over the products, preferring higher quality products to inferior ones. Thus, if prices were the same, the consumers would all choose to buy the top quality good. In this type of market the demand is directly affected by the properties of consumers' income distribution. If consumers have different incomes and thus, different willingness to pay for higher quality products, firms can profitably split the market by offering products differentiated in qualities at different prices. Therefore, in vertically differentiated markets, income inequality among consumers becomes a key factor in determining the product varieties offered by the firms.

The purpose of this paper is to study the effect of income inequality on market outcomes in vertically differentiated markets, with particular interest in the range of qualities on offer. Many countries have experienced significant increases in income inequality over the past several decades.¹ The welfare implications of higher income inequality have been analyzed by looking at the consumer expenditures data and measuring the corresponding change in consumption inequality (Krueger and Perri [13], Jappelli and Pistaferri [11]). Data on expenditures do not take into account the changes in quality of products consumed, and these are endogenous to the consumer demand and depend on the distribution of consumer incomes. This paper uses a stylized model to demonstrate that firms' decisions on product characteristics are affected by the degree of inequality and these choices have important welfare implications.

The line of research linking income distribution of the consumers to the industry structure dates back to Gabszewicz and Thisse [9], and has been cultivated by them [10] as well as by Shaked and Sutton [18], [19], [20]. These authors demonstrate that the interplay of the industry cost structure and demand conditions, which are the

¹See Caminada and Goudswaard [6], Atkinson [3].

outcome of the underlying income distribution, determine the degree of concentration and the maximum number of firms in vertically differentiated markets (Shaked and Sutton [20]). They have almost nothing to say, however, about what kind of products these firms would be producing.

Endogenous quality choices in duopolies with uniform distributions of consumer preferences for quality are analyzed by Motta [15], Lehmann-Grube [14] and Aoki and Prusa [2]. Motta [15] studies two types of duopolistic markets, one with price and the other with quantity competition. He finds quality differentiation in equilibrium in both Bertrand and Cournot setting, with larger quality spreads under Bertrand. This result holds under two different cases of fixed and variable costs of quality improvement. Lehmann-Grube [14] demonstrate that in duopolistic markets the top quality firm makes a higher profit for any convex fixed-cost function of quality, and under the scenarios of simultaneous or sequential choice of qualities. Aoki and Prusa [2] study the effect of simultaneous versus sequential quality choices on the equilibrium quality levels under the assumption of quadratic fixed costs of quality.

Multiproduct competition has been analyzed by De Fraja [8], who considers a vertically differentiated industry with an exogenous number of firms that simultaneously choose both qualities and quantities of their products, and a general distribution function for consumer preferences with upper and lower bounds on incomes. Johnson and Mayatt [12] study optimal quality choices of a multiproduct monopoly in response to entry by another firm and how these choices are affected by the properties of the distribution of consumer preferences for quality. The authors, however, do not endogenize the number of firms in the market.

The paper most closely related to this one is Benassi, Chirco and Colombo [4]. These authors analyze the effect of income concentration on product differentiation and obtain solutions for quality and pricing decisions of duopolistic firms. To obtain analytical results they assume that consumer incomes are distributed with a trapezoid distribution, and that the market is not covered. The authors find that more

concentrated income distributions lead to more product differentiation. In this paper I propose to further this research agenda by modifying the existing models to make them applicable for studying the effects of changes in the consumers' income distribution on the firms' entry decisions and the optimal choices of product attributes and prices for a lognormal income distribution function. I solve the model numerically to obtain the equilibrium number of firms in the market, the qualities they produce, and the prices they charge.

The baseline theoretical model is due to the Shaked and Sutton [18]. Firms compete in a three stage non-cooperative game by making entry, product quality and pricing decisions. Each firm, if enters, supplies a single product variety, and consumers can choose to purchase at most one good. The outputs of the model are the number of firms in the market, product qualities and prices, and the major input is the income distribution of the consumers. Shaked and Sutton [18] assume that the income distribution is uniform and obtain analytical solution for a duopoly. Changes in the degree of income inequality can be modeled with a uniform distribution by shifting its endpoints. However, the support of the distribution would change, also altering the nominal scale of the market. Since the demand functions depend on nominal incomes, the uniform distribution cannot be used to analyze the purely redistributive effects of changes in income inequality on firms' decisions.² This paper models the distribution of consumer incomes with a lognormal distribution, which has been found to provide the most accurate fit of real-life income distributions among other candidates for parametric estimation (Pinkovskiy and Sala-i-Martin [17]).

The most valuable insight from the present analysis is that income inequality among consumers affects the intensity of competition. The result that greater income inequality enables more firms to enter the industry with positive market shares dates back to Gabszewicz and Thisse (1979) and has been replicated in most of the works that followed. In this paper I am also able to demonstrate that income inequality

²Similar argument is made in Benassi, Chirco and Colombo [4] to motivate the use of trapezoid income distribution.

impacts the degree of product differentiation in the market. Low degree of heterogeneity in consumer incomes intensifies price competition in the last stage of the game, thus, in order to soften it, firms differentiate their products more when income inequality is lower. Greater inequality in consumer incomes reduces the incentive to differentiate and intensifies quality competition among firms for the shrinking middle and higher-income sections of the market. Thus, when income inequality is higher, firms locate their products in higher ranges of the quality spectrum, closer to each other, raising the average product quality and decreasing the degree of product differentiation. Competition between more similar products tends to reduce their prices. However, when income inequality is very high, the top quality producer chooses to serve only the rich segment of the market, and the low price elasticity of demand of these consumers allows him to charge a higher price.

The model predicts that aggregate consumer welfare is higher in economies with greater income inequality. Higher intensity of quality competition in these economies induces lower-quality firms to raise the quality of their products and offer these products at lower prices. Thus, the majority of consumers are better off when income variability is high. Greater income inequality also decreases the degree of product differentiation; therefore, on a quality-adjusted basis, consumption inequality may be lower in economies with a higher degree of income inequality.

The main results of the paper are derived under the assumption of the costless quality choice when there exists an upper bound on the best quality that can be produced. This simplifies the analysis and makes it possible to study product differentiation as the outcome of a purely demand-driven strategic behavior. However, this assumption is limiting since it makes the quality choice of the top quality producer trivial. Thus, I also compute the model for the case when the cost of quality improvement is fixed and quadratic and show that the results hold when the burden of quality improvement falls primarily on fixed costs and these costs rise sufficiently fast in quality. The assumption of quickly diminishing returns, especially at very

high levels of quality, is realistic for many industries where quality improvements are achieved via investments in R & D.³

The paper is organized as follows. After describing the model in Part 2, I outline the solution method in Part 3. The discussion in this part also includes the issues of existence and uniqueness of equilibria. Part 4 of the paper gives the results of the model. Part 5 concludes.

2 The Model

The analysis here follows very closely that of Shaked and Sutton (1982). The economy is inhabited by two kinds of agents: consumers and firms. The firms produce distinct, substitute goods, that are differentiated by quality. Consumers are heterogeneous in income and have preferences over the goods produced by the firms, with the ordering of preferences being identical for all consumers. They can choose to purchase only one good, basing their decision on the choice of qualities they face and prices, or make no purchase. These decisions generate demand functions for the firms, who face a more complicated oligopolistic competition problem.

Each of the firms produces only one good. They compete in a three-stage non-cooperative game. In the first stage each of the firms chooses whether it would enter the market. In the second stage, upon observing the number of entrants, firms that have entered the industry choose the specifications of their product, that is, its quality. In the last stage firms observe both the number and quality choices of their rivals and set their prices.

The game is solved using Subgame Perfect Nash Equilibrium concept, beginning at the last stage of the game and moving up the game tree.

Stage 3. Choosing Optimal Prices.

³For example, see Alexander, Flynn and Linkins [1] for evidence of diminishing returns to R & D in the pharmaceutical industry. Some discussion of other cost of quality structures and their implications for the results is provided in the concluding part of the paper.

Denote the number of firms that have entered the industry in stage 1 of the game by N . These firms produce distinct, substitute goods. Each firm $k = 1, \dots, N$ produces a good of quality k . Denote the quality level of firm k 's product by u_k . These u_k 's have been chosen at stage 2 of the game and at the current stage are common knowledge. Assume these qualities are ordered $u_0 < u_1 < \dots < u_N \leq \bar{u}$, where u_0 is the quality of the outside good. For the baseline case of zero costs to quality improvement, assume also that there is an exogenous upper bound on quality \bar{u} , that is, $u_N \leq \bar{u}$. This assumption is not made when the costs of quality are fixed and quadratic. Also, let the price of the outside good be $p_0 = 0$. Each firm k is choosing the price of its product p_k .

The economy is inhabited by a continuum (measure one) of consumers identical in tastes but heterogeneous in income. Each consumer has income t which is distributed with a cdf F with support on $[0, \infty)$. Consumers purchase only one good or make no purchase and consume an outside good $k = 0$. For every consumer good k is characterized by the level of utility she obtains from consuming good k , which is assumed to be equal to u_k , and price of this good p_k . The preferences of consumer with income t from consuming good (u_k, p_k) are described by utility function⁴

$$U(t, (u_k, p_k)) = u_k (t - p_k). \quad (1)$$

Define the income level t_k such that a consumer with income t_k is indifferent between purchasing good k at price p_k and good $k - 1$ at price p_{k-1} . That is,

$$U(t_k, (u_k, p_k)) = U(t_k, (u_{k-1}, p_{k-1}))$$

⁴This utility function is the same as in Shaked and Sutton [18]. The important assumption on preferences is that consumer's willingness to pay for quality is increasing in income. The degree of inequality in consumer incomes would then result in the same qualitative implications for the demand for different qualities and, thus, similar conclusions about the effects of income inequality on equilibrium outcomes. In fact, one of the main results of greater product differentiation in the economies with lower degree of inequality has been also established by Benassi, Chirco and Colombo [4] for the case of a duopoly and a trapezoid distribution of consumer preferences, when consumer's utility function is quasi-linear as in Mussa and Rosen [16].

for $k = 1, \dots, N$. Then

$$t_k = \frac{u_k p_k - u_{k-1} p_{k-1}}{u_k - u_{k-1}}. \quad (2)$$

In this stage of the game the firms simultaneously choose their prices so as to maximize their profits taking as given the prices of their rivals.

The profit of firm $k = 1, \dots, N - 1$ is

$$\Pi_k = p_k [F(t_{k+1}) - F(t_k)] - C(u_k), \quad (3)$$

the profit of firm $k = N$ is $\Pi_N = p_N [1 - F(t_N)] - C(u_N)$, and $C(u_k)$ is the fixed cost function of producing quality u_k , $k = 1, \dots, N$.

Each firm $k = 1, \dots, N$ solves $\max_{p_k \geq 0} \Pi_k$. The solution is the best response function (possibly, a correspondence) of firm k

$$p_k^{BR} = p_k(p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_N; u_1, \dots, u_N).$$

The Nash equilibrium of the pricing stage of the game is the set of price functions $\{p_k^{NE}(u_1, \dots, u_N)\}_{k=1, \dots, N}$ that solves the system of equations formed by the best response functions of all firms.

Stage 2. Choosing Optimal Qualities.

In this stage of the game firms observe the number of entrants N and simultaneously choose the quality of their own product u_k , $k = 1, \dots, N$.

Each firm solves:

$$\max_{u_k > u_0} \{p_k^{NE} [F(t_{k+1}^{NE}) - F(t_k^{NE})] - C(u_k)\} \quad (4)$$

where $p_k^{NE} = p_k^{NE}(u_1, \dots, u_N)$ and $t_k^{NE} = \frac{u_k p_k^{NE} - u_{k-1} p_{k-1}^{NE}}{u_k - u_{k-1}}$. The equilibrium of this stage of the game is a vector of qualities (u_1^*, \dots, u_N^*) , where u_k^* is firm k 's best response

to $u_{-k}^* = (u_1^*, \dots, u_{k-1}^*, u_{k+1}^*, \dots, u_N^*)$ for all $k = 1, \dots, N$. Denote by Π_k^* the maximized value of profits of firm k , $k = 1, \dots, N$, at (u_1^*, \dots, u_N^*) . The equilibrium qualities and profits depend on the number of entrants in stage 1 of the game N .

Stage 1. Entry.

Denote by ε the entry cost for any firm k . If a firm chooses to enter this market it can expect to make $\Pi_k^*(N)$. Thus, a firm will enter if $\Pi_k^*(N) - \varepsilon \geq 0$. The number of firms in the market N^* is a Nash equilibrium if $\Pi_k^*(N^* + 1) - \varepsilon < 0$ for some k . That is, the entry of an additional firm would lead to some firms making negative profits net of the entry cost.

In what follows the entry cost ε is assumed to be very small, so as to get the maximum possible number of entrants in the market. That is, N^* is considered to be an equilibrium number of firms if $\Pi_k^*(N^* + 1) = 0$ for some k .

3 Solving the Model

In this section of the paper I discuss the computational algorithm and assumptions made in order to obtain the numerical solution of the model.

3.1 Assumptions

3.1.1 Consumers' Income Distribution and Income Inequality

The consumers' income distribution is assumed to be lognormal⁵ with cdf $F(\mu, \sigma)$. Since the purpose of the paper is to study the effect of income inequality on firms' decisions, parameters μ and σ are chosen so as to make the variance of the income

⁵The lognormal distribution is often used to model the real world income distributions. The present computer code can be easily modified for another specification of the distribution function. It is important to keep in mind, though, that the choice of a different distribution function may affect the existence and uniqueness (or multiplicity) properties of the solution.

distribution vary, while keeping the mean income constant. Denote the mean of the income distribution by A .

The standard measure of income inequality is the Gini coefficient. The Gini coefficient is a number between 0 and 1, with higher values corresponding to greater income inequality. According to the United Nations Development Programme's "Human Development Report 2006", it ranges from 0.19 in Azerbaijan to 0.74 in Namibia with an average of about 0.4 for the 126 countries in the report.⁶ The Gini coefficient can be calculated for a given continuous cdf function as ⁷

$$G = 1 - \frac{1}{A} \int_0^\infty (1 - F(y))^2 dy.$$

For given μ and σ of the lognormal distribution the corresponding Gini coefficient can be computed using the formula above. The parameter σ is allowed to vary from 0.34 to 1.6. For each value of σ from this range, the value of the parameter μ is then chosen so as to keep the mean of the distribution constant at the chosen value for A . With these specifications the Gini coefficient varies from 0.19 to 0.74, which corresponds to the maximum range observed in the data.

3.1.2 Parameters Choice

To compute the model numerically it remains to specify the values for the mean income A and the quality of the outside good u_0 . For the model with zero costs of quality, the upper bound on quality \bar{u} also needs to be specified. For the model with fixed and quadratic costs of quality improvement, assume that the cost function $C(u_k) = cu_k^2$, $c > 0$, $k = 1, \dots, N$. Part 4 of the paper contains the results that have been obtained for $A = 15$, $u_0 = 1$, $\bar{u} = 10$, and $c = 0.01$. Robustness tests have been

⁶According to the report, examples of countries with low income inequality include Denmark, Japan, and Sweden, all with Gini coefficients around 0.25 in 2006. In Europe Turkey has the highest measure of income inequality at 0.44. US has a Gini coefficient of 0.41, Canada - 0.33, and Mexico - 0.5.

⁷See Dorfman [7] for derivation.

performed to verify that different values of these parameters affect the quantitative, but not the qualitative predictions of the model.

3.2 Computational Algorithm

The issues of existence and uniqueness of equilibria for these types of models are typically not considered in the literature due to their extreme difficulty. Instead, the focus is on studying the characteristics which equilibria must have, if they exist. When looking for a numerical solution of the model, however, it is very important to know whether it exists and, if so, whether it is unique. The model here has multiple stages, and existence and uniqueness problems may arise at each of them. Unfortunately, due to the complexity of the problem the analytical proofs are not feasible for any part of the game. I turn to the numerical methods to verify existence and uniqueness or multiplicity of equilibria.

The computer code used to solve the model has been written with an explicit goal of making it possible to verify at any stage of the game that what is being found as a solution is in fact an equilibrium and, if so, whether there are other equilibria besides the one being computed.

The model is solved using Matlab software. The procedure is repeated for different values of μ and σ to study the effects of changes in income distribution function parameters on the model outcomes. For each value of N stages 2 and 3 of the model are written as functions. The stage 3 function takes as given the vector of qualities (u_1, \dots, u_N) and produces the vector of Nash equilibrium prices $(p_1^{NE}(u_1, \dots, u_N), \dots, p_N^{NE}(u_1, \dots, u_N))$. This function is called upon in the body of the stage 2 function, which, for a particular value of N , attempts to compute the Nash equilibrium qualities (u_1^*, \dots, u_N^*) .

For each set of values of μ and σ , the procedure begins with the monopoly case, that is, $N = 1$. The number of firms is increased until at least one of the firms is making nonpositive profits when the qualities and prices of all firms constitute Nash

equilibria of the respective stages of the game. Here is a brief outline of the procedure:

- I. Specify parameters μ and σ of the income distribution function.
- II. Let the number of firms in the market be $N = 1$. Use a stage 2 function to compute optimal quality of the monopolist and stage 3 function to find the profit-maximizing price. Verify that the profit is positive.
- III. Let $N = N + 1$. Call a stage 2 function for N which seeks to find the Nash equilibrium qualities (u_1^*, \dots, u_N^*) . This function uses the stage 3 function to compute Nash equilibrium prices for any distribution of firms' qualities. Compute profits for all firms.
- IV. If at least one of the firms is making nonpositive profits, conclude that the equilibrium number of firms is $N^* = N - 1$ and the equilibrium qualities and prices are as found for the case of $N - 1$ firms. Otherwise, go back to step **III**.

Next I discuss the algorithms for computing stage 2 and 3 equilibria in greater detail, also addressing the issues of their existence and uniqueness.

3.2.1 Stage 3: Computing Optimal Prices

The input of the stage 3 function is a vector of firms' qualities (u_1, \dots, u_N) . Each firm $k = 1, \dots, N$ optimally chooses its price p_k so as to maximize its profit, taking the prices of other firms $p_{-k} = (p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_N)$ as given. For a given vector p_{-k} this is a simple single-variable constrained optimization problem, and the best response function can be computed as $p_k^{BR} = p_k(p_{-k})$. Any point where the best response functions of all firms $k = 1, \dots, N$ intersect is a Nash equilibrium of this stage of the game. Visual tests conducted for different income distribution specifications, $N = 2, 3, 4$, and various combinations of qualities (u_1, \dots, u_N) lead to the conclusion that the point of intersection exists and is unique.

The visual tests involve plotting the price best response functions of firms to see if they intersect at a single point. In order to use two-dimensional graphs when $N > 2$, I use the following procedure. For $N = 3$, let $p_1^{BR} = f_1(p_2, p_3)$, $p_2^{BR} = f_2(p_1, p_3)$, and $p_3^{BR} = f_3(p_1, p_2)$. For each value of p_1 , find p_3 that solves $p_3 = f_3(p_1, f_2(p_1, p_3))$, that is, with price of firm $k = 2$ as a best response to prices of firms $k = 1$ and $k = 3$. Similarly, for each value of p_3 , find p_1 that solves $p_1 = f_1(f_2(p_1, p_3), p_3)$. Then plot thus obtained price best response functions of firms $k = 1$ and $k = 3$ to check for the intersection point. A similar procedure can be used to verify the existence and uniqueness of equilibrium in prices for $N > 3$.

I use the method of simple iterations on best response functions to find this unique Nash equilibrium. This method is the most simple and reliable. It can be slower than the alternative methods, but unreliability of other methods in this case prevents their meaningful use.⁸ The solution in the price-setting stage does not depend on the assumptions made about the costs of quality, which is not the case for the stage 2 of the game.

3.2.2 Stage 2: Computing Optimal Qualities

For a given N the stage 2 function searches for an equilibrium vector of qualities (u_1^*, \dots, u_N^*) . Notice that each particular vector (u_1^*, \dots, u_N^*) corresponds to $N!$ equilibria in terms of the identities of the firms. To illustrate, suppose that two firms X and Y enter the market at stage 1. If there is an equilibrium with firm X producing u_1^* and firm Y producing u_2^* , then there is also an equilibrium with Y producing u_1^* and X producing u_2^* . For all purposes here these symmetric equilibria are considered to be identical and are treated as one equilibrium. Thus, when looking for equilibria with two firms producing distinct qualities I will assume that one of the firms is producing

⁸An alternative solution method would involve solving the system of first-order conditions. The more efficient numerical methods for solving systems of nonlinear equations are based on replacing the problem with that of minimizing a functional. The surface of this functional turns out to have a very irregular shape due to the assumption of the lognormal probability density function. As a result, the solution obtained using these methods is very sensitive to the initial guess.

the lower quality good while the other one is making the higher quality one, and they both know their respective positions. For $N = 3$ the respective quality positions for each of the firms are "fixed" at low, middle, and high. There is a similar preassigned ordering for larger N .

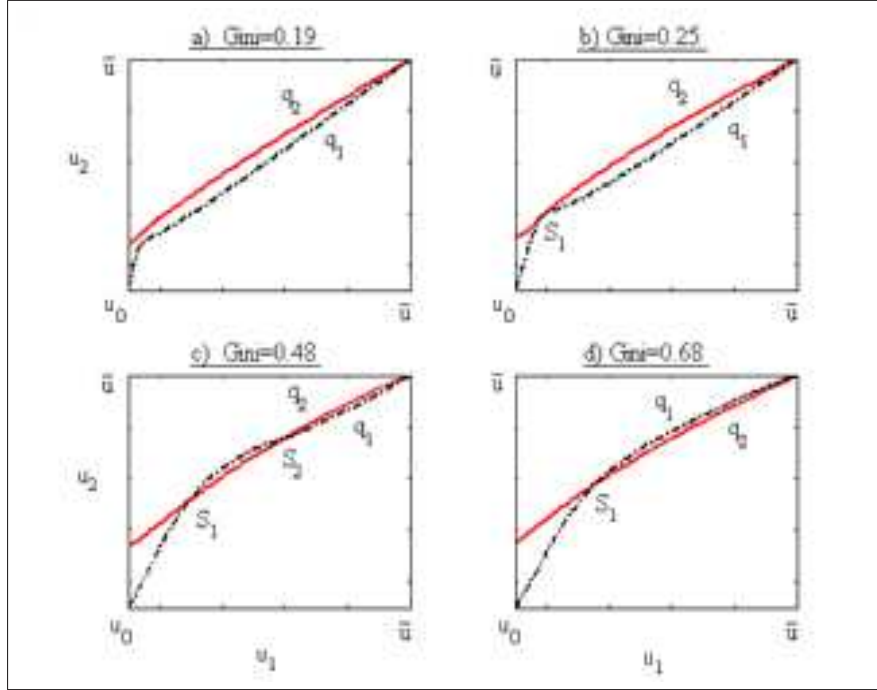
Zero costs of quality

In the model with no costs to producing higher quality the top quality firm's best response to any quality choices by its rivals is $u_N = \bar{u}$. For $N = 2$ the problem at this stage is a simple one of finding u_1 that maximizes firm 1's profit, taking as given $u_2 = \bar{u}$ and the price functions from stage 3 ($p_1^{NE}(u_1, \bar{u}), p_2^{NE}(u_1, \bar{u})$). For $N = 3$ the procedure is looking for an intersection point of the quality best response functions for firms 1 and 2 when $u_3 = \bar{u}$ and the price functions from stage 3 are ($p_1^{NE}(u_1, u_2, \bar{u}), p_2^{NE}(u_1, u_2, \bar{u}), p_3^{NE}(u_1, u_2, \bar{u})$). Denote the quality best response function of firm 1 by $u_1 = q_1(u_2, \bar{u})$ and that of firm 2 by $u_2 = q_2(u_1, \bar{u})$. Below I plot four examples, each for an economy with a different value of the Gini coefficient, illustrating four possible situations for equilibria in this stage of the game.

Note that when costs are zero and $N > 2$, there is always an equilibrium with all firms producing the top quality \bar{u} . Suppose two firms choose qualities \bar{u} . The price competition between them means that both firms charge zero prices in equilibrium and earn zero profits. No other firm can benefit by choosing a different, lower quality, since it cannot also charge a lower price than its higher quality competitors so as to gain a positive share of the market. That is, if two or more firms produce \bar{u} , the Bertrand competition at stage 3 ensures that all firms earn zero profits in equilibrium, and no single firm has an incentive to deviate from producing \bar{u} .

In Figure 1 a) the quality best response functions of firms 1 and 2 do not intersect in any point besides the one where they both produce \bar{u} . When this is the case, there does not exist an equilibrium with firms producing differentiated products. The best response functions in Figure 1 b) and d) coincide in one other point besides (\bar{u}, \bar{u}) ,

Figure 1: Equilibria in stage 2 of the game, $N = 3$, zero costs of quality



point S_1 . The conclusion in these two cases is that the equilibrium with desired properties exists and is unique. In Figure 1 c) the best response functions intersect in two other points, S_1 and S_2 , where $u_1, u_2 \neq \bar{u}$. Thus, potentially there are two solutions with firms producing distinct qualities. The equilibrium at S_2 cannot be computed by any procedure involving iterations, since it is non-stable. The code uses the simple iterations methods to compute the quality choices corresponding to S_1 . The quality vector with thus chosen u_1 and u_2 is the solution for this stage of the game.

Similar graphs can be obtained for $N > 3$. Let $N = 4$ and denote the quality best response functions of the three lower quality firms by $u_1 = q_1(u_2, u_3, \bar{u})$, $u_2 = q_2(u_1, u_3, \bar{u})$, and $u_3 = q_3(u_1, u_2, \bar{u})$. For each (u_3, \bar{u}) let u_1 be the solution to $u_1 = q_1(q_2(u_1, u_3, \bar{u}), u_3, \bar{u})$ obtained by the method of simple iterations. Similarly, u_3 solves $u_3 = q_3(u_1, q_2(u_1, u_3, \bar{u}), \bar{u})$ for every (u_1, \bar{u}) . Denote these solutions by $\hat{u}_1 =$

$\hat{q}_1(u_3, \bar{u})$ and $\hat{u}_3 = \hat{q}_3(u_1, \bar{u})$. The problem becomes that of finding an intersection of functions $\hat{q}_1(u_3, \bar{u})$ and $\hat{q}_3(u_1, \bar{u})$, if it exists. This task is analogous to the one described for the case of $N = 3$ above. A similar procedure can be used for $N > 4$.

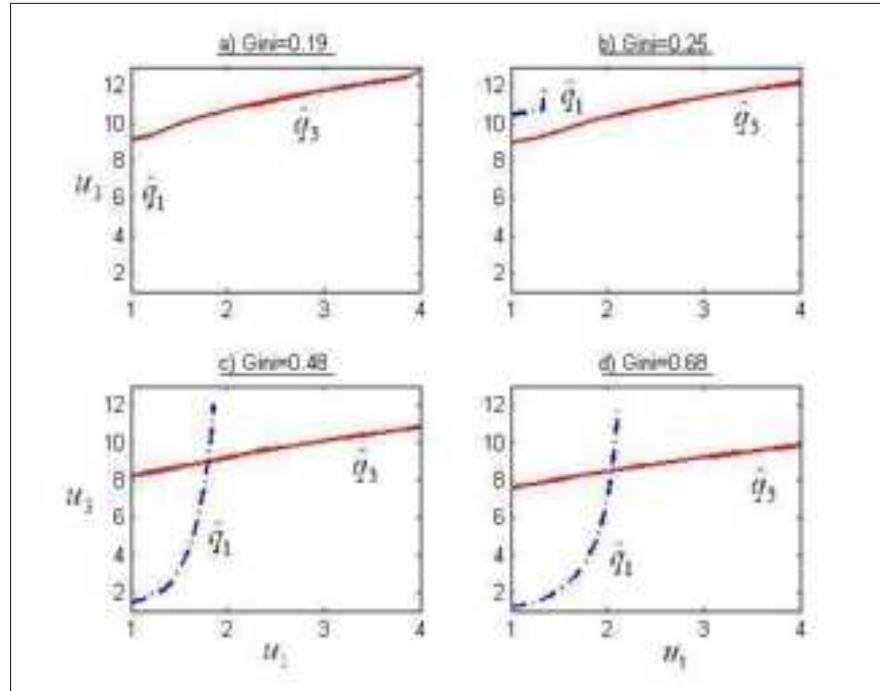
Quadratic fixed costs of quality

With fixed costs of quality improvement, the equilibrium with all firms producing the same quality does not exist. Also, the choice of the highest quality producer is no longer trivial, and the value for top quality depends on the properties of the consumer demand and the choices of rivals. Visual tests conducted for different specifications of the income distribution and different number of firms show that equilibrium, if it exists, is unique. As before, for $N = 2$ visual tests involve plotting the quality best response functions of firms $k = 1, 2$ to check for the intersection points. For $N > 2$, the best response functions of firms $k = 1$ and $k = N$ are solved for and plotted, and the decisions of all other firms are also optimal, as in the procedure described above. Below I take the same four economies as in the case of zero costs of quality improvement and plot the quality best response functions for $N = 3$:

Figures 2 a) and b) show that when the value of the Gini coefficient is low at 0.19 or 0.25, there does not exist an equilibrium with three firms in the market. Firm $k = 1$ produces $u_0 = 1$; in the quality-setting stage this is equivalent to shutting down.⁹ For each of the two economies in Figures 2 c) and d) there is only one intersection point, that is, there is a unique equilibrium.

⁹No firm can produce a quality below u_0 and gain a positive share of the market since the price of the outside good is zero.

Figure 2: Equilibria in stage 2 of the game, $N = 3$, fixed quadratic costs of quality



4 Results

Number of firms

The degree of income dispersion, measured by the Gini coefficient (G), determines the equilibrium number of firms in the market. Intuitively, if all consumers had the same income and the same willingness to pay for better quality, at most one firm would be able to survive in the market. The higher quality firm can always price any competitor out of the market, since for any price it gets the demand from either all or none of the consumers.¹⁰ With some degree of inequality, the price-setting stage of the game is no longer "all or nothing". The top quality firm may find it more profitable to set a higher price and get only the demand from affluent consumers to

¹⁰As long as the average income and the cost of quality are such that a firm can produce a quality above u_0 , charge a price that would prevent any lower quality firm from profiting if it gets all of the demand with a lower price, and make a positive profit. Otherwise, no firms would enter the market if entry costs are nonzero.

gaining the whole market with a low price. Thus, the higher the degree of inequality, the more firms can survive in the market with positive market shares.¹¹

The results of both versions of the model show that in the economies with the value of the Gini coefficient below some cutoff value, only two firms choose to enter the market, that is, $N^* = 2$. When the costs are zero, this cutoff value of the Gini coefficient is equal to 0.2492. For the model with fixed quadratic costs of quality it is equal to 0.2686. Economies with values of G above the threshold are inhabited by consumers whose incomes are distributed less equally. Greater degree of consumer heterogeneity gives the firms more "room" to compete. As a result, up to three firms can enter the market in these economies and earn positive profits and the equilibrium number of firms is $N^* = 3$. Thus, income inequality determines the number of firms that can coexist in a vertically differentiated industry with positive market shares, with more firms inhabiting the markets in less egalitarian economies.¹²

Equilibrium qualities and prices

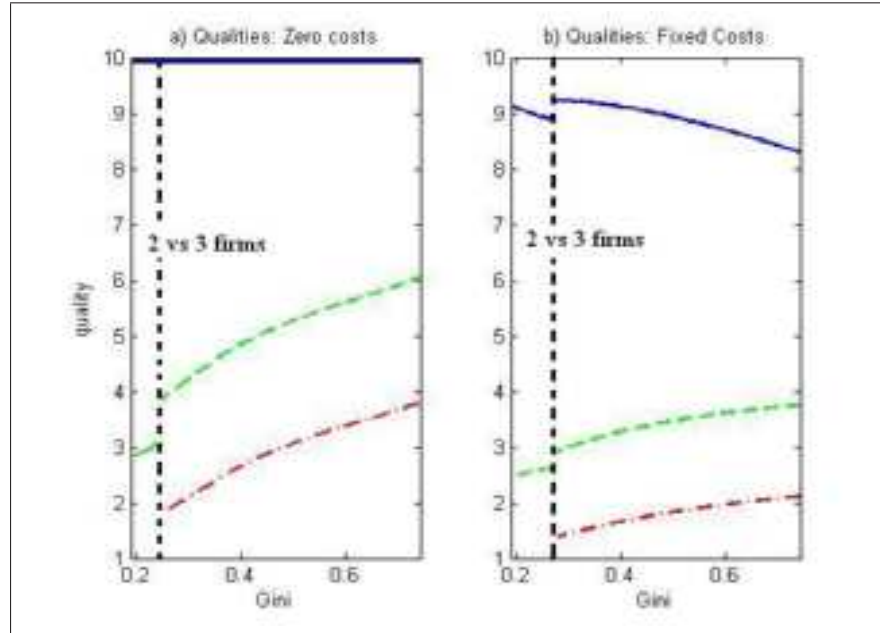
In both versions of the model, with zero costs of quality and when the costs are fixed and quadratic, the results show that the degree of product differentiation declines in inequality while the average product quality increases.¹³ Figure 3 below illustrates these findings. The dotted vertical lines drawn at $G = 0.2492$ for the model with no costs and at $G = 0.2686$ for the model with fixed quadratic costs of quality separate the cases for $N^* = 2$ and $N^* = 3$.

¹¹For a uniform distribution of incomes, Gabszewicz and Thisse [9] were the first to establish that significant income dispersion is necessary for more than one firm to survive in the market. Shaked and Sutton [18] and [19] also show that the upper bound on the number of firms depends on the spread of the income distribution.

¹²In the model with fixed quadratic costs of quality, the exact number of firms depends on the value of the cost function parameter c , since it affects firms' profits. The number, however, still increases with the degree of inequality.

¹³Benassi, Chirco and Colombo [4] also find that more concentrated incomes imply larger product differentiation in a model with two firms, trapezoid distribution of consumer incomes, quasi-linear utility function, and zero costs of quality.

Figure 3: Firms' qualities



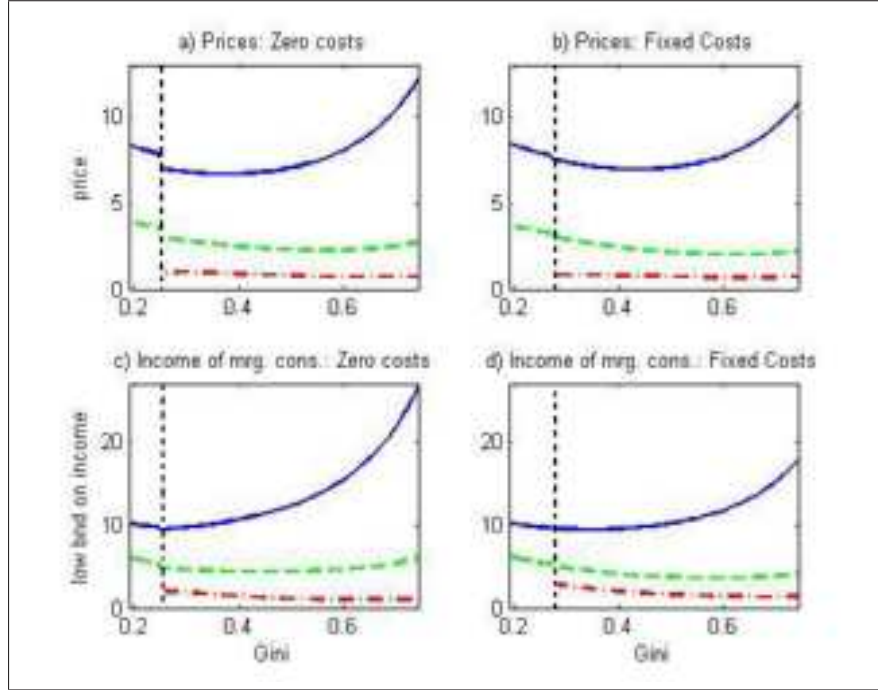
Note that in the model with no costs to producing higher quality the top quality firm always chooses to produce the highest possible quality $\bar{u} = 10$. When the costs of quality are fixed and rise steeply, the quality of the top firm is slightly lower in the economies with more unequal income distributions for the same equilibrium number of firms. The revenues of the top quality producer decline as its market share shrinks in competition with other firms for higher-income consumers, so it finds it more profitable to lower the quality in order to reduce costs (See Figure 6).¹⁴

Figures 4 a) and b) give equilibrium price decisions of firms for two versions of the model. As before, the dotted vertical lines separate the cases for $N^* = 2$ and $N^* = 3$. In both models, the equilibrium prices of firms producing the lowest and the second highest quality products are lower in the economies with higher levels of

¹⁴This finding may not be robust to the specification of the fixed cost of quality function. If the cost function is not very steep, the top quality firm may choose to increase its quality and differentiate itself more from the rivals. Here I present the findings for the model with no costs and the model with fixed and quadratic costs of quality. The predictions of the latter model are robust as long the cost function is sufficiently steep. I discuss the implications of other assumptions about the costs of quality in the concluding part of the paper.

consumer income inequality. The price of the top quality product is decreasing at first, and then becomes an increasing function of G for the values of this parameter above some threshold value.

Figure 4: Firms' prices and Incomes of marginal consumers



Figures 4 c) and d) depict the income levels of the marginal consumers, t_k 's. Recall that a consumer with income t_k is indifferent between purchasing from firm k and $k - 1$. Thus, for example, in economies where three firms enter the market, the demand for the top quality firm is given by the fraction of population with incomes above t_3 , the consumers with incomes between t_2 and t_3 buy from the second highest quality firm, those with incomes between t_1 and t_2 purchase the good of the lowest quality, and the rest choose not to buy and consume the outside good.

The baseline model with zero costs of quality is best suited for the analysis of the effects of demand on firms' optimal choices. To this end, consider four hypothetical economies, each characterized by a different value of the income inequality measure G ,

and plot the respective income distribution functions and equilibrium market shares of firms for the case when quality improvement is costless.

Figure 5: Income density functions in Economies 1 through 4 and market shares of firms: Zero costs

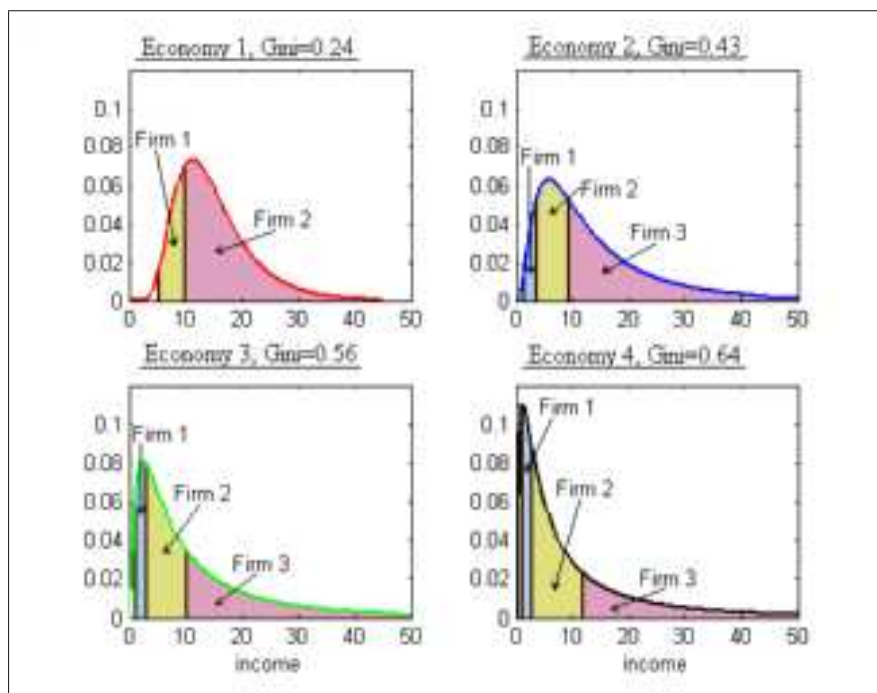


Figure 5 gives the consumer income distributions for each of these four economies. The economies are ordered by the degree of inequality in the consumer incomes, with Economy 1 inhabited by consumers with the most equal distribution of incomes. The vertical lines mark the income levels of the marginal consumers, t_k 's, and the shaded areas of the graphs represent the demands for each of the firms or, equivalently, their market shares.

As G increases, the income distribution becomes more skewed to the right. The most prevalent type of consumer (the income distribution peaks at her income level) becomes increasingly more poor from Economy 1 to Economy 4, while the fraction of consumers with incomes in the middle range is rapidly shrinking. The income density functions in the economies with greater income inequality are characterized

by thicker tails, which means that these economies also have more consumers with incomes above the mean.

Consumers with higher incomes constitute the more attractive market for the firms, since for each level of quality more affluent consumers have higher willingness to pay. In the economies with a more egalitarian distribution of incomes the most attractive market for the firms is composed of the middle income consumers, since they are the most prevalent type. Low variability of incomes in this group results in small differences in willingness to pay for higher quality. This allows the top quality producer to capture most of the market by pricing low enough to keep its inferior quality competitor serving the segment of relatively poor consumers.

The result of higher product differentiation in the economies with lower levels of inequality is due to the intensity of price competition in the last stage the game. Greater homogeneity of consumer incomes leads to more intense price competition. Its effects can only be mitigated through greater degree of product differentiation. The number of firms is also lower, since if more than two firms were to enter in the Economy 1, they would not be able to locate far enough from each other in the quality spectrum in stage 2 of the game to sufficiently lessen the intensity of price competition in stage 3.

Greater income inequality increases the variability of incomes of the consumers in the more attractive, higher income part of the market. The second highest quality firm can now benefit by increasing the quality of its product without causing a knock-out price competition in the last stage of the game. Thus, the result of higher quality choices of lower and medium quality firms in the economies with more unequally distributed incomes is due to both the desire to chase the shrinking fraction of consumers with higher incomes and the reduced incentive to differentiate the products.

Figure 3 and Figures 4 a) and b) show that the quality of the second highest quality good increases and the prices of two higher quality firms decline until the middle income market becomes too small for both of the firms to share, and the highest

quality good producer "gives up" these middle class consumers to serve exclusively the rich.

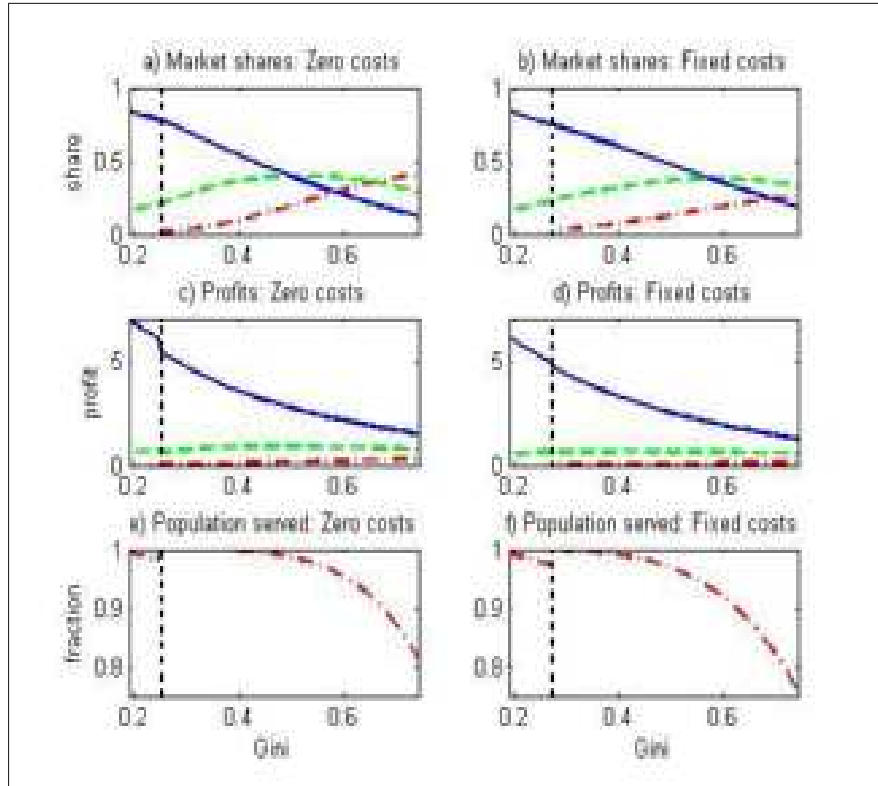
In Figure 3 the quality of the second highest quality good is increasing in G , that is, in the degree of income inequality. In Figures 4 a) and b) the price of this good is decreasing in G , while the price of its higher quality competitor is "U" - shaped. The equilibrium price of the top quality product begins to increase in economies with very high levels of income inequality because the consumers purchasing it are so affluent that their demand is inelastic for higher values of prices. Figures 4 c) and d) also show that the marginal consumer of the top quality firm (t_3) is becoming increasingly richer after some value of G .

Other results: Market shares, profits, and welfare

Additional results are demonstrated in Figures 6 and 7. Figures 6 a) through d) give market shares and profits of firms for the two versions of the model. Increases in income inequality induce the low quality firm to produce better quality product and charge lower price. Combined with the increase in the proportion of the relatively poor consumers in the population, this leads to greater market share and higher profits for the low quality firm. The market share and profits of the top quality firm decrease in the level of income inequality of the consumers. Greater inequality of incomes results in more intense quality competition between the two top quality producers, enabling the second highest quality firm to steal some business from its top quality competitor. The shrinking middle class eventually leads to the decline in the second highest quality producer's market share as well. The market shares of all firms get closer to each other in size as the income distribution becomes more unequal, causing the concentration to fall with greater degree of income inequality (Figures 7 a) and b)).

Figures 6 e) and f) show the total fraction of consumers in the market that choose to purchase from one of the firms as a function of the degree of inequality in incomes.

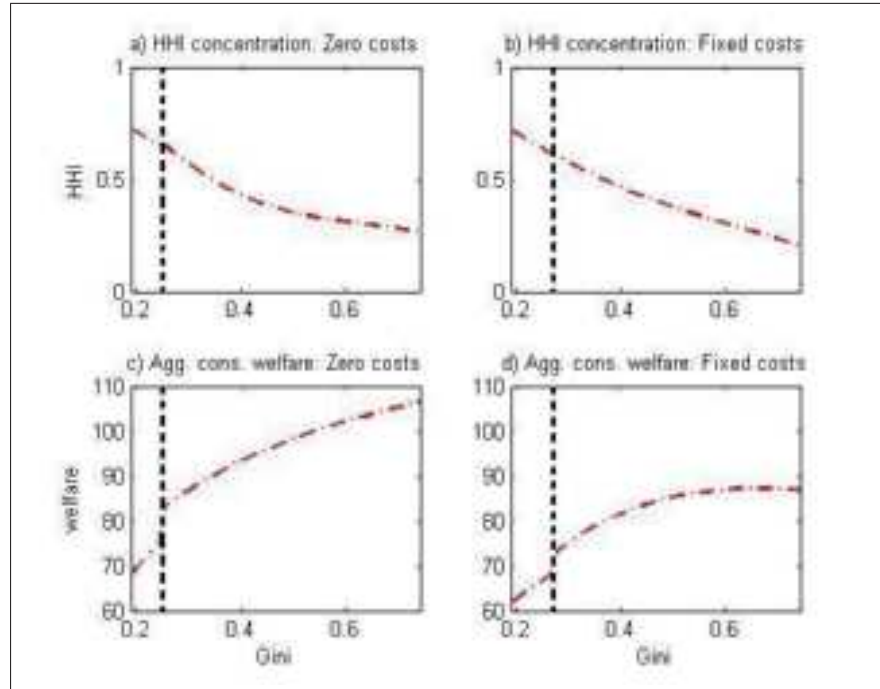
Figure 6: Additional results: market shares, profits, and market coverage



Observe that at the threshold value of the Gini coefficient, when an additional firm chooses to enter the market, the market is almost covered. Further increases in the income inequality measure are manifested in larger fraction of the consumers with low incomes, who cannot be induced to buy even the lowest quality good, notwithstanding its lower price and better quality.¹⁵ The consumers that do end up purchasing from one of the firms benefit substantially from the more intense price and quality competition among the firms that accompany increases in income inequality. The increase in their welfare is greater than the corresponding decline in the welfare or lower income consumers, thus, the aggregate consumer welfare is higher in economies with greater degree of income inequality (Figures 7 e) and d)).

¹⁵Yet, no other firm would choose to enter the market to serve these consumers with a low quality since it would loose in the price competition with its higher quality competitors.

Figure 7: Additional results: concentration and consumer welfare



5 Conclusion

In this paper I study how income inequality among consumers affects the decisions of firms operating in vertically differentiated industries. The model used to address this question makes the following important assumptions: a) each consumer chooses at most one good out of a variety of products differentiated in quality; b) consumers have different incomes, and richer consumers are willing to pay more for better products; c) the products are supplied by firms that compete by choosing qualities and prices in a non-cooperative three-stage game, with each firm supplying only one type of quality; and d) there are either no costs to producing higher quality products or these costs are fixed and quadratic. In order to study the effects of changes in income inequality on model outcomes, I assume a lognormal distribution for consumer incomes and solve the model numerically, holding the mean of the distribution constant while changing the variance.

The results demonstrate that income inequality impacts the degree of product differentiation in vertically differentiated markets. The industries in the economies with greater income inequality are characterized by a greater number of firms and more intense quality competition. The following results hold under the assumption of zero costs of quality and when costs of quality are fixed and quadratic: 1) the degree of product differentiation declines in inequality and 2) the average quality is higher in the economies with less equal distributions of income.

The strictly convex fixed cost of quality function implies that developing a product of better quality is costly and increasingly so. This assumption may not be valid for some industries where the fixed costs of quality improvement are less steep. Intuitively, the second result should be robust to any specification of the fixed cost of quality function, since for lower quality firms the incentive to attract higher income consumers becomes stronger with higher income inequality while the incentive to differentiate their product weakens. However, when the fixed cost function is not very steep, the top quality firm may choose very high quality when income distribution is unequal, differentiating itself strongly from the rivals, resulting in wider quality gap. Thus, both results will hold as long as the fixed costs of quality are sufficiently steep.

The model also assumes zero variable costs to quality improvement. Shaked and Sutton [20] show that when the burden of quality improvement falls primary on fixed costs, that is, the unit variable costs do not increase much with quality, then the number of firms in the market will be limited. This occurs because higher quality producers can use price competition to undercut their lower quality rivals and possibly drive them out of the market entirely. When the variable costs rise sufficiently fast in quality, however, higher quality firms may no longer find it profitable to undercut lower quality firms by lowering prices. Thus, more firms can survive in the market with positive market shares.¹⁶ The questions are 1) how would the number

¹⁶Berry and Waldfogel [5] also make this argument in their empirical investigation of how market size affects the level of top quality on offer and product concentration in vertically differentiated industries.

of firms be affected by the degree of consumer income inequality? and 2) which parts of the quality spectrum would fill up faster in the economies with varying degrees of consumer heterogeneity? While this paper does not aim to provide rigorous investigation of these issues, some hypotheses can be made. Convex variable unit costs of quality weaken the price competition, but do not eliminate it entirely. Thus, firms have the incentive to differentiate their products more in economies with lower degree of consumer heterogeneity when the costs of quality are variable and convex, and the number of firms is higher in economies with greater income inequality. What types of products would these firms chose to produce? Greater income inequality means more very poor and very rich consumers, and less consumers with the incomes in the middle range. Intuitively, sections of the market with relatively more consumers make it possible for more firms to survive even when prices are close to marginal costs. Thus, for two economies with the same average income and size of the market, we would expect to see greater concentration of products at the ends of the quality spectrum in the economy with higher inequality of consumer incomes. Whether the average quality on offer would decline or increase with inequality is unclear.

The model assumes that consumers' preferences are represented by a Cobb-Douglas utility function. However, as long as the preferences are such that consumers with higher incomes have higher willingness to pay for better products the qualitative predictions of the model should remain unaltered, for the intuition behind them remains the same: when consumers tastes for quality are more homogeneous the price competition is more intense. Thus, in order to weaken it, firms differentiate there products more, and fewer firms can survive in the market with positive market shares. Likewise, Benassi, Chirco and Colombo [4] use a quasi-linear utility function as in Mussa and Rosen [16] and find that in a duopoly the quality spread is larger when the consumers are more homogeneous.

Another result of the paper is lower prices in the economies with higher levels of inequality. Lower degree of product differentiation leads to more intense price

competition, pushing down the prices of all firms in the market. However, in the economies where income inequality is very high, the top quality producer chooses to serve only the rich consumers; their demand is more price inelastic, which enables him to charge a higher price. Also, market shares and profits of all firms are distributed more equally in less egalitarian economies, and the consumers are better off in terms of aggregate welfare.

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